# Slope Fields and Differential Equations

An equation like $\frac{dy}{dx} = y \ln x$, that contains a derivative is a differential equation. To determine **y** as a function of **x** when we are given its derivative and the value of the function at a given point is called an **initial value problem**. To **solve a differential equation** means to find the unique equation that satisfies the given conditions among the family of equations with the given derivative.

We can use graphical representations with slope fields and Euler's Method to identify the unique solution. The best way to understand slope fields is to draw some by hand. To do this, we draw small segments of tangents lines at selected points. We can do this because a differentiable function is locally linear at the point of tangency and can be approximated by its tangent line over a small interval.

1. Given the function: $y = 0.5x^2$. Write the derivative: _____

At each grid point, calculate the value of the derivative and draw a short line segment with that slope.

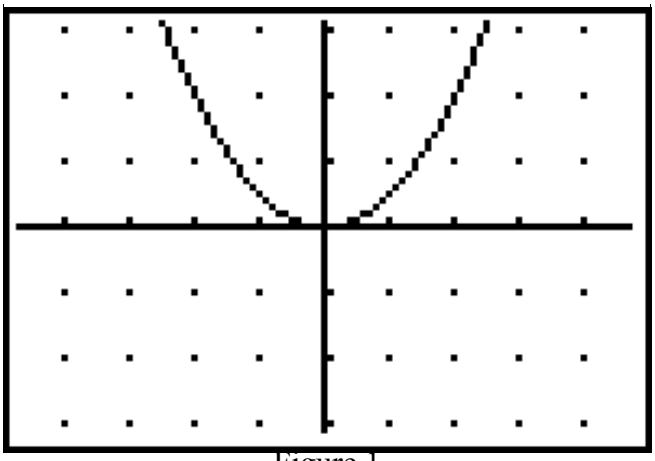


Figure 1

What family of functions seems to match all the slope fields?

What is an initial condition of the function graphed?

2. Sketch a function that matches this slope field.

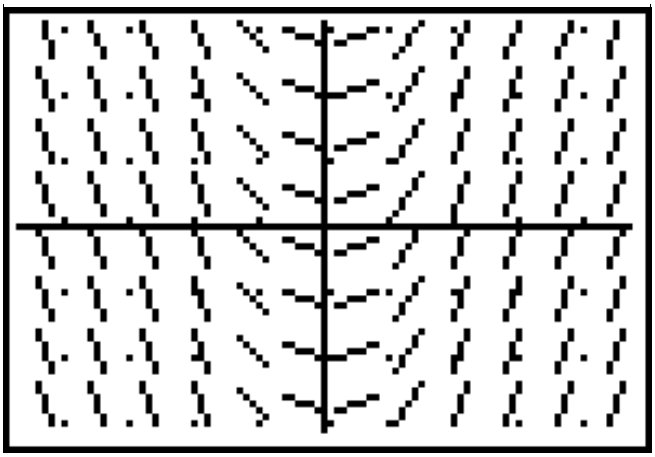
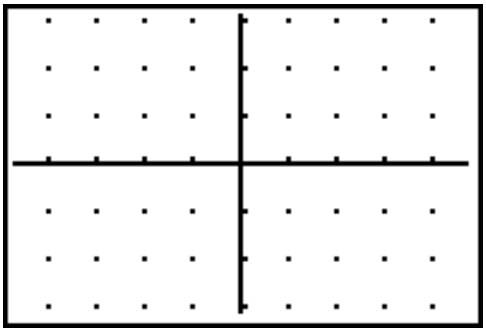


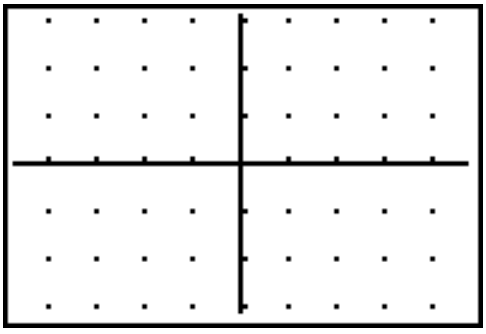
Figure 2

What family of functions seems to match this slope field?

3. If $\frac{dy}{dx} = 2x$, sketch the slope field



4. If $\frac{dy}{dx} = \frac{1}{x+3}$, sketch the slope field.



5.

a) Sketch the path of the unique solution if the graph passes through (0,1).

b) Sketch the path of the unique solution if the graph passes through (0,-1).

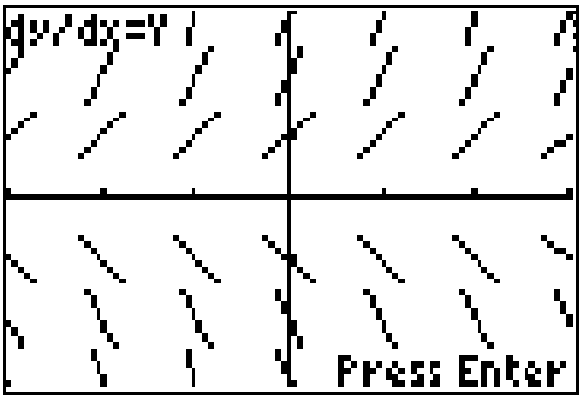



**Figure 5**

c) What familiar functions do these two graphs resemble?

d) Given: $\frac{dy}{dx} = y$ , verify your guess analytically

6. Figure 6 shows the slope field for the differential equation $\frac{dy}{dx} = \frac{-4x}{9y}$.

a) Calculate the slope at the point (2,-1). Mark this point on the graph. Does the calculated slope seem reasonable? Explain.

b) Start at the point (0,2) and draw a graph that represents the particular solution of the differential equation that contains that point. Where does the curve seem to go after it touches the x-axis? What geometric shape does the figure seem to be?

c) Solve the differential equation algebraically. Find the unique solution that contains the point (0,2). Does this graph agree with your guess in part b)?

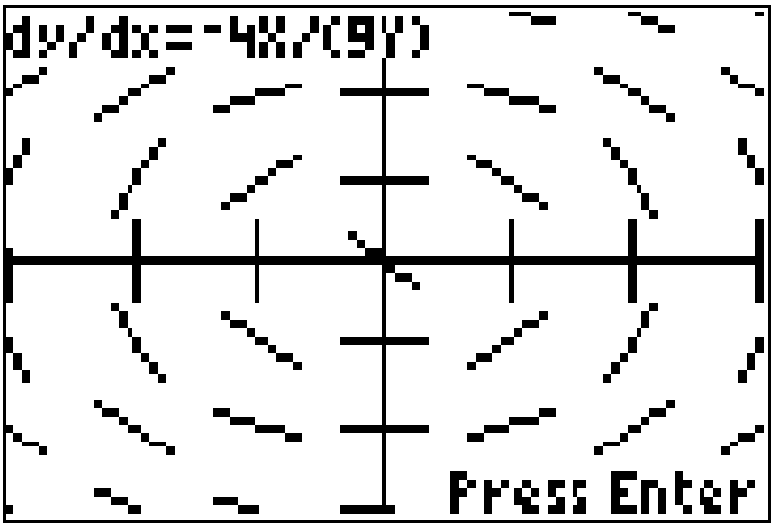
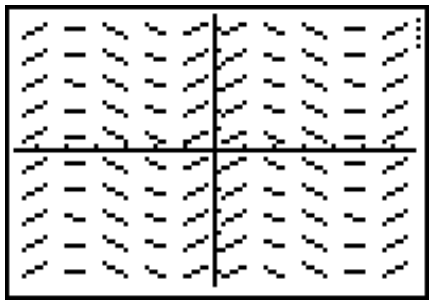
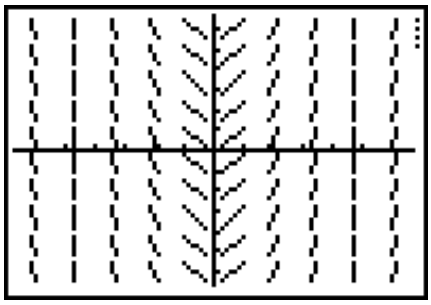
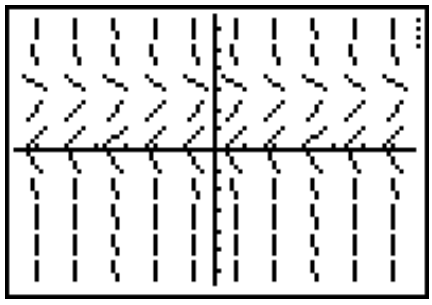


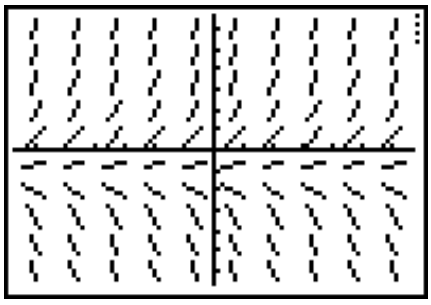

**Figure 6**

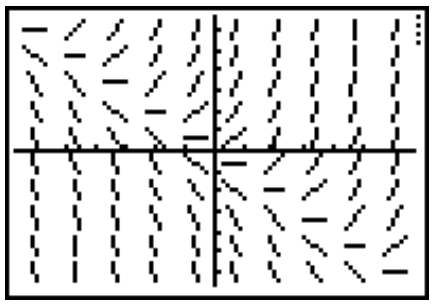
Below are six examples of **slope fields**. Match them with the correct differential equation. Explain each choice.

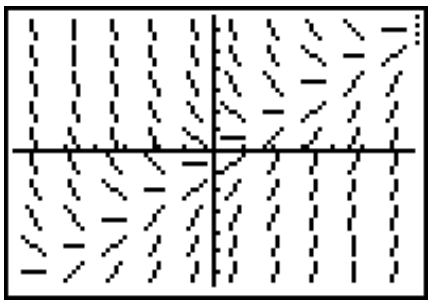
a. 

b. 

c. 

d. 

e. 

f. 

1. $\frac{dy}{dx} = x - y$

2. $\frac{dy}{dx} = 1 + y$

3. $\frac{dy}{dx} = \cos x$

4. $\frac{dy}{dx} = 2x$

5. $\frac{dy}{dx} = x + y$

6. $\frac{dy}{dx} = y(3 - y)$